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EXPLICIT FORM OF THE OPTIMUM CONTROL LAW FOR A RIGID AIRCRAFT FLYING IN A TURBULENT ATMOSPHERE

G. Coupry

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16. Abstract Flight of military aircraft at high speeds and low altitudes makes it necessary to use ride control systems to improve comfort, handling qualities and combat ability. In designing such systems, flexibility can be omitted, due to the large difference between the frequencies associated with flight mechanics and those associated with the first flexible mode. Closed loop systems which feed back some output information to the controls through appropriate filters are widely used by designers, but increase the response time of the aircraft during maneuvering, which can be a source of difficulty in some missions. The open loop system described here senses turbulence, which is used, after filtering, to activate the controls. This type of system has no effect on the handling qualities of the aircraft. The paper explains how Wiener's theory can be used to derive the explicit form of the transfer function of the filter used for control.

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Notation /9-2*

$a(t)$: unit pulse response of a parameter $q(t)$ to the reduced turbulence $\frac{w}{v}$.

$w(t)$: vertical component of the atmospheric turbulence.

v: flight speed (level).

$\beta(t)$: deflection of a control surface.

$b(t)$: unit pulse response of $q(t)$ to deflection β.

$k(t)$: unit pulse response of the control system.

*: symbol for the product of convolution.

$z(t)$: altitude of aircraft.

M: mass of aircraft.

I: rotational inertia during pitch.

ℓ, s: reference length and area.

ρ: air density.

θ: attitude angle.

α: angle of attack.

C_L: dimensionless lift coefficient.

C_m: dimensionless moment coefficient.

$C_{L,\chi}$ and $C_{m,\chi}$: their derivatives with respect to χ.

$$\xi = \frac{z}{\ell}$$

$$c = \frac{I}{\rho/2\, s \ell C_{m,\alpha}}$$

$$m = \frac{M}{\rho/2\, s \ell C_{z,\alpha}}$$

$$\eta = \frac{C_{m,q}}{C_{m,\alpha}}$$

*Numbers in the margin indicate pagination in the foreign text.

$\lambda = \frac{C_{z,\beta}}{C_{z,\alpha}}$

$\nu = \frac{C_{m,\beta}}{C_{m,\alpha}}$

$\omega_R = \frac{\omega \ell}{V}$ reduced frequency.

$h = \frac{\mathcal{D}}{2C}$

$\xi^2 = \frac{\nu - \lambda}{\lambda c}$

$\nu_R^2 = \frac{1}{c} + \frac{2h}{m}$

$\alpha_R = \frac{1}{2\nu_R}\left(2h + \frac{1}{M}\right)$

$A(i\omega)$: transfer function to turbulence.

$B(i\omega)$: transfer function to control surface commands.

$K(i\omega)$: control transfer function.

$S_{W(\omega)}$: spectral density of turbulence.

L: scale of turbulence.

$r = \frac{L}{\ell}$.

σw^2: variance of turbulence.

$\tilde{\sigma}_{\ddot{Z}}{}^2$: variance of the response of the aircraft without automatic pilot.

$\sigma_{\ddot{Z}}{}^2$: variance of the response of the aircraft with automatic pilot.

$\gamma = \frac{\tilde{\sigma}_{\ddot{Z}}{}^2}{\sigma_{\ddot{Z}}{}^2}$: gain due to control system. /9-3

$F(\omega_R)$: weighting law used to filter high frequencies.

s: Laplace variable.

s_k: poles of the reduced control law.

Introduction

A number of missions required of military aircraft now involve low-altitude, high-speed conditions. Since turbulence is especially heavy and frequent close to the ground, the flight crew is consequently exposed to severe conditions which, when added to the problems of following the terrain, may cause marked fatigue, lowered reflexes, and in extreme cases, unfitness for combat. In addition, there may be severe problems of structural fatigue in the airframe, which is subjected to high load factors. This latter point has been a special concern of the Structures and Materials Panel, which has performed some research on this problem as part of its investigation of the impact of CCV systems on structural behavior.

The following discussion is intended to show the conditions under which a military aircraft (a delta-wing aircraft, for example) can be equipped, in a relatively simple manner, with an automatic control system which will decrease its response to turbulence. The system described makes use of conventional control surfaces and in no way changes the flight mechanics of the aircraft in the absence of turbulence. The study described here deals only with the longitudinal behavior of a rigid aircraft in the presence of vertical turbulence, for small motions in the vicinity of cruising conditions. There would be no problem, however, in extending the conclusions to lateral behavior. After a discussion of the various possible approaches (closed-loop or open-loop system), the Wiener theory [1, 2] is used to express the explicit form of a filter which, based on a turbulence measurement performed in real time on board the aircraft, delivers to the control surface orders by which it is possible to minimize the variance of a given response -- acceleration at the center of gravity, for example. The law clearly shows the effect of speed and density, and consequently the self-adaptive characteristics of the automatic pilot. Finally, the gains which may be generated by this system are also stipulated, and the effects of various parameters (scale of turbulence, mass of aircraft, etc.) are analyzed.

I. Critique of the Different Optimization Systems

Two main types of system may be considered for flight control in turbulence: closed-loop systems, which feed back certain responses of the aircraft to the control surfaces after appropriate filtering, and open-loop systems, which adjust the control surfaces on the basis of a measurement of the turbulence encountered by the aircraft.

In the first system (Fig. 1a), the entire behavior of the aircraft is modified, especially its flight mechanics, its handling qualities, etc. In the second (Fig. 1b), all flight properties and qualities remain unchanged, with the exception of the turbulence transfer functions. This is due to the fact that in closed-loop systems, the set of differential equations representing the motion of the aircraft is modified by the occurrence of a "feedback" designed to damp the various modes of the aircraft, while in open-loop systems this set of equations remains unchanged, the second member alone being transformed by the presence of the automatic pilot.

NASA and Boeing (Wichita Division) first performed a number of studies on flight optimization in turbulence by a closed-loop system under the LAMS Program, followed more recently by the more general CCV System Program [3, 4]. In all cases, the basic principle was the use of a negative feedback grid attacking a complex system of control surfaces on the basis of accelerometric and gyrometric measurements on the airframe. The grid constants were selected so as to make a favorable change in the turbulence transfer functions of certain critical responses.

This study was followed up and completed, with flight tests, on a B52 specially equipped with horizontal and vertical canards for the test. The control system was theoretically designed with allowances for 30 elastic modes and checked on an analog computer to take the efficiency limitations of the control surfaces into account. The use of high-gain, wide-passband servo controls necessitated especially complex stability testing. Flight tests were performed on the aircraft itself only after the theoretical analysis had been confirmed by wind tunnel tests on a model of similar dynamic characteristics equipped with the system. Nine hours of real turbulent flight were performed, during which the turbulence transfer functions with and without control system were measured. Fig. 2, drawn from Ref. [4], gives the transfer functions for one flight condition. The gain obtained through use of the system, that is, the ratio of the variance of the uncontrolled aircraft to that of the controlled aircraft, was on the order of 3.

The excellent results obtained come under a very general American policy oriented toward the systematic use of active flight controls, both in turbulence and in order to increase critical buffeting speeds or reduce handling loads. Extensive use of these techniques is already being made in the Boeing supersonic transport project, and the B1 bombardier will definitely be equipped with a turbulence flight control system.

One might ask why there should still be any interest in open-loop systems in view of these developments. There are three main reasons, in our opinion:

1. The need to equip an aircraft using a closed-loop system with special control surfaces in most cases. /9-4

2. The limitations of these systems, which, in the area of flight mechanics, are related to handling qualities (there is no way of damping the pitch mode to infinity).

3. Problems of stability related to negative feedback.

Although points 1 and 3 require no further comment, a striking illustration for point 2 is provided by the fact that it is possible to eliminate all responses of a rigid aircraft to turbulence by the use of an open-loop system without changing its flight mechanics, provided that two infinitely efficient independent control surfaces are available. Obviously, this is a trivial and highly simplified example, its purpose being merely to show the possibility of results which could not be achieved by any closed-loop system.

Let us consider a rigid aircraft controllable by two independent control surfaces: a conventional elevator of deflection β, and a direct lift control of deflection σ. With the notation given at the beginning of this report, the longitudinal behavior of the aircraft is represented by the equations:

$$\begin{cases} M\ddot{z} = \frac{\rho}{2}SC_{z,\alpha}\, v^2\alpha + \frac{\rho}{2}Sv^2C_{z,\beta}\beta + \frac{\rho}{2}Sv^2C_{z,\sigma}\sigma \\ I\ddot{\theta} = \frac{\rho}{2}S\ell v^2C_{m,\alpha}\alpha - \frac{\rho}{2}S\ell vC_{m,q}\dot{\theta} - \frac{\rho}{2}S\ell v^2C_{m,\beta}\beta - \frac{\rho}{2}S\ell v^2C_{m,\sigma}\sigma \end{cases} \tag{1}$$

where α is the angle of attack, related to the attitude θ and the vertical gust velocity w by the kinematic equation:

$$\alpha = \theta - \frac{z'}{V} + \frac{W}{V} \tag{2}$$

Let us assume the turbulence to measured in real time on board the aircraft by analog solution of Eq. (2), and that the two control surfaces are given control commands in the respective forms:

$$\begin{cases} \beta = \mu_0 \frac{w}{v} \\ \sigma = \mu_1 \frac{w}{v} \end{cases}$$

Introducing these terms into Eq. (1) with allowances for Eq. (2), one obtains the system:

$$\begin{cases} M\ddot{z} + \frac{\rho}{2}SvC_{z,\alpha}z' - \frac{\rho}{2}Sv^2C_{z,\alpha}\theta' = \frac{\rho}{2}Sv(C_{z,\alpha} + \mu_0C_{z,\beta} + \mu_1C_{z,\sigma})w \\ I\ddot{\theta} + \frac{\rho}{2}S\ell^2vC_{m,q}\dot{\theta} + \frac{\rho}{2}S\ell v^2C_{m,\alpha}\theta + \frac{\rho}{2}S\ell vC_{m,\alpha}z' = \frac{\rho}{2}S\ell v(C_{m,\alpha} + \mu_0C_{m,\beta} + \mu_1C_{z,\sigma})w \end{cases} \tag{3}$$

The first member of the equations, which represents the flight mechanics of the aircraft, is the same with or without automatic pilot, while all responses of the aircraft to turbulence can be completely cancelled by choosing for μ_0 and μ_1 the solutions to the equations:

$$\begin{cases} C_{z,\alpha} + \mu_0 C_{z,\beta} + \mu_1 C_{z,\sigma} = 0 \\ C_{m,\alpha} + \mu_0 C_{m,\beta} + \mu_1 C_{m,\sigma} = 0 \end{cases}$$

which completely cancel the second member of System (3).

This is merely an academic example, since the control surfaces have been assumed infinitely efficient, the aircraft completely rigid, and the aerodynamic forces independent of the reduced frequency.

These reservations apart, the example provides a perfect illustration of the attractiveness of open-loop systems: it is theoretically possible to cancel any response of the aircraft to turbulence, but without changing the flight mechanics of the aircraft in any way.

Unfortunately, the range of application of these systems is limited to the frequencies associated with the flight mechanics of the rigid aircraft. This is because we have implicitly assumed from the beginning that there is "a turbulence w" responsible for the responses of the aircraft which can be measured at a given point on the structure. On an absolute level, this assumption is in contradiction to that of isotropy, according to which turbulence is no more likely to be uniform in a spanwise direction than in the direction of flight. Under these conditions, there is a risk that point measurement of the turbulence will not provide any information on the field of disturbance actually encountered by the aircraft.

A recent study [5] permits a closer view of this problem by establishing the frequency below which a local turbulence datum will provide significant information on the gust field encountered by the aircraft. For this purpose, the half-span b of the aircraft is compared with the transverse coherence distance of the turbulence:

$$\Lambda = 1.403 \frac{V}{\omega}$$

computed for a Karman spectrum at a velocity of translation V. It can be seen that whenever the length Λ associated with the velocity V and a pulsation mode ω is much greater than the span 2b, it may be assumed that the waves are constant over the entire span. On the other hand, if the parameter

$$p = \frac{\Lambda}{b} = 1.403 \frac{V}{b\omega}$$

is on the order of magnitude of unity or less, the assumption of constant spanwise waves is no longer valid, and a point measurement of turbulence will provide no information on the turbulence encountered by the aircraft as a whole. Table 1, which gives the values of the parameter p for the pitch mode and the first bending mode of four recent aircraft, clearly shows that although a local turbulence datum will be significant within the frequency range of the flight mechanics, it loses all value at the frequencies associated with the distorsion modes.

The above considerations appear to provide a basis for some provisional conclusions and a philosophy of design for turbulent flight optimization systems. If the distorsion modes have a predominant effect on the response of the aircraft, only closed-loop systems can be used with success, since they do not require a knowledge -- illusory in any case -- of the field of turbulence encountered by the aircraft. If, on the other hand, the flight mechanics is the main factor responsible for aircraft behavior in turbulence, open-loop systems are much more attractive, since they permit control without modification of the handling qualities /9-5 of the aircraft (assumed to be ideal), and since they can be constructed within the frequency range considered. In mixed cases, it might be possible to use open-loop control for low frequencies, with the addition of a negative feedback system for the high frequencies associated with the distorsion modes. In this case the quality of the open-loop system would be evaluated within the frequency range for which it is designed.

II. Open-Loop Optimization

II.1. Statement of Problem

As we have seen in the preceding section, a rigid aircraft can be almost completely optimized by an open-loop system if two independent control surfaces are available (conventional elevator and direct lift control). Although some reservations must be made due to the oversimplification of the problem, this system is probably the best conceivable under these conditions.

The problem to be dealt with now is more complex: the equipping of current aircraft, possessing conventional control surfaces, with an open-loop automatic control system which minimizes the variance of the responses to turbulence. The specific aircraft which will be considered is a delta-wing airplane (the Mirage III). Improved turbulent control of this aircraft is desired through the manipulation of conventional elevons initiated by signals derived from on-board turbulence measurements in real time.

The problem may be expressed mathematically as follows:

"a(t) and b(t) being the unit pulse responses to reduced turbulence w/v and to the deflection β of the control surface, respectively, to find a physically realizable function k(t) (that is, the unit pulse response of a stable system) such that the control surface commands:

$$\beta(t) = k(t) * \frac{w(t)}{v}$$

<u>minimize the variance of the response</u>:

$$q(t) = a(t) * \frac{w(t)}{v} + b(t) * \beta(t)$$

<u>of the parameter considered</u>."

It is assumed that the aircraft is totally rigid, that the point measurement w(t) of the turbulence is representative of the field encountered by the aircraft at all frequencies, and that the aerodynamic stresses are independent of the reduced frequency. The variances of the responses are computed for the frequency bands within which the turbulence is assumed to be uniform over the span of the aircraft.

II.2. Transfer Functions of the Aircraft

Neglecting any "screening" phenomenon, we will write the longitudinal flight mechanics equations for a rigid aircraft, linearized for cruising conditions at a speed v, in the form:

$$\left\{ \begin{array}{l} M\ddot{z} = \frac{\rho}{2} S v^2 C_{z,\alpha}\, \alpha + \frac{\rho}{2} S v^2 C_{z,\beta}\, \beta \\ I\ddot{\theta} = -\frac{\rho}{2} S \ell v^2 C_{m,\alpha}\, \alpha - \frac{\rho}{2} S \ell^2 v C_{m,q}\, \dot{\theta} - \frac{\rho}{2} S \ell v^2 C_{m,\beta}\, \beta \end{array} \right. \tag{4}$$

Taking into account the kinematic equation

$$\alpha = \theta - \frac{z'}{v} + \frac{w}{v}$$

and with the notation given at the beginning of this report, there is no problem in obtaining the nondimensional form:

$$\left\{ \begin{array}{l} m\frac{\ell^2}{v^2}\ddot{\xi} + \frac{\ell}{v}\dot{\xi} - \theta = \frac{w}{v} + \lambda\beta \\ -\frac{\ell}{v}\dot{\xi} + c\frac{\ell^2}{v^2}\ddot{\theta} + \eta\frac{\ell}{v}\dot{\theta} + \theta = -\frac{w}{v} - \nu\beta \end{array} \right. \tag{5}$$

A(iω) and B(iω) will be termed the transfer function of the acceleration $\ddot{z}$ to reduced turbulence $\frac{w}{v}$ and the transfer function to the control surface commands β, respectively (these are the Fourier transforms of a(t) and b(t), respectively).

With the introduction of the reduced frequency:

$$\omega_R = \frac{\omega \ell}{V}$$

these transfer functions can be written in the form:

$$A(i\omega) = \frac{V^2}{\ell_m} A'(i\omega_R) \; ; \; B(i\omega) = \frac{\lambda V^2}{\ell_r} B'(i\omega_R)$$

where $A'(i\omega_R)$ and $B'(i\omega_R)$ are expressed solely as functions of the reduced variables:

$$\begin{cases} A'(i\omega_R) = \dfrac{\omega_R^2 - 2ih\omega_R}{\omega_R^2 - 2i\omega_R \nu_R \omega_R - \nu_R^2} \\[2ex] B'(i\omega_R) = \dfrac{\omega_R^2 - 2ih\omega_R + \xi^2}{\omega_R^2 - 2i\omega_R \nu_R \omega_R - \nu_R^2} \end{cases} \qquad (6)$$

II.3. Formulation of Optimization Problem

$K(i\omega)$ will denote the Fourier transform of $k(t)$, that is, the transfer function of the control law. Given these conditions, the expression for the transfer function of the automatically controlled aircraft to turbulence is:

$$T(i\omega) = A(i\omega) + B(i\omega)K(i\omega)$$

that is, by introducing the reduced control law:

$$\tilde{K}(i\omega_R) = \lambda K\left(i \frac{V}{\ell} \omega_R\right) \qquad (7)$$

the expression:

$$T(i\omega) = \frac{V^2}{\ell_m} \left(A'(i\omega_R) + B'(i\omega_R)\tilde{K}(i\omega_R)\right)$$

The spectral density $\phi_{\ddot{z}}(\omega)$ of the acceleration at the center of gravity is therefore expressed, as a function of the spectral density $s_w(\omega)$ of the turbulence, as:

$$\phi_{\ddot{z}}(\omega) = \frac{1}{V^2} |T(i\omega)|^2 S_w(\omega)$$

and the variance $\sigma_{\ddot{z}}^2$ of the response, at the frequencies for which it has been defined, will be:

$$\sigma_{\ddot{z}}^2 = \int_0^{+\infty} \phi_{\ddot{z}}(\omega) F(\omega)\, d\omega = \frac{V^2}{\ell^2 m^2}\int_0^{+\infty} |A'(i\omega_R) + B'(i\omega_R)\tilde{K}(i\omega_R)|^2 F(\omega) S_w(\omega)\, d\omega$$

where $F(\omega)$ is an imaginary filter selected so as to limit the integral domain on which the variance is computed.

It can now be seen that, given the Taylor assumption, the turbulent spectral density models can be written in the form:

$$S_w(\omega_R) = \frac{\tau}{\pi}\sigma_w^2\, \psi_w(\omega_R)$$

where r is the ratio of the scale L of turbulence to the reference length ℓ of the airplane. As a result, for Dryden's spectral model:

$$\psi_w(\omega_R) = \frac{1 + 3\omega_R^2\tau^2}{(1 + \omega_R^2\tau^2)^2}$$

and for Karman's model:

$$\psi_w(\omega_R) = \frac{1 + 8/3\,(1.339\,\tau\omega_R)^2}{(1 + (1.339\,\tau\omega_R)^2)^{11/6}}$$

Taking these observations into account, the variance $\sigma_{\ddot{z}}^2$ of the response can be written in the form:

$$\sigma_{\ddot{z}}^2 = \frac{\tau V^2 \sigma_w^2}{\pi \ell^2 m^2}\int_0^{+\infty} |A'(i\omega_R) + B'(i\omega_R)\tilde{K}(i\omega_R)|^2 \psi_w(\omega_R) F(\omega_R)\, d\omega_R \tag{8}$$

and the problem of optimization will be formulated thus:

<u>To find a physically realizable transfer function $\tilde{K}(i\omega_R)$ such that:</u>

$$\delta\int_0^{+\infty} |A'(i\omega_R) + B'(i\omega_R)\tilde{K}(i\omega_R)|^2 \psi_w(\omega_R) F(\omega_R)\, d\omega_R = 0 \tag{9}$$

<u>the extremum, of course, being a minimum.</u>

Stated in this form, the optimization problem does not depend explicitly on flight speed. Consequently, it will result in a reduced control law $\tilde{K}(i\omega_R)$ which is independent of v. This law will depend only on the air density ρ, the scale of turbulence L, and the dimensionless coefficients $C_{L,\chi}$ and $C_{m,\chi}$ (and, through these coefficients, on the Mach number).

Another conclusion may be drawn from determination of the gain:

$$\chi = \frac{\tilde{\sigma}_{\ddot{z}}^2}{\sigma_{\ddot{z}}^2}$$ [partly missing in original]

considered as the ratio of the variance of the response of the aircraft without automatic pilot ($\tilde{\sigma}_{\ddot{z}}^2$) to the variance ($\sigma_{\ddot{z}}^2$) of the response of the aircraft with automatic pilot. From the fact that: /9-7

$$\tilde{\sigma}_{\ddot{z}}^2 = \frac{\tau v^2 \sigma_w^2}{\pi \ell^2_m{}^2} \int_0^{+\infty} |A'(i\omega_R)|^2 \phi_w(\omega_R) F(\omega_R) d\omega_R$$

it may be deduced that the gain also does not depend explicitly on flight speed, but only on ρ, L, and the $C_{L,\chi}$ and $C_{m,\chi}$ parameters.

II.4. Expression of Control Law, Gain and Effect of Parameters

Based on Eq. (9), which formulates the optimization problem, the reduced control law $\tilde{K}(i\omega_R)$ can be computed through the use of the Wiener filter theory. This method, used for the first time by J. Boujot [6], is preferred over the temporal approach of the Karman filter method, which presents considerable difficulties due to the fact that the turbulence correlations and aerodynamic stresses must be expressed as solutions to differential equations.

Since this method has been described in detail in a recent article, only the principal findings will be given here. Taking

$$F(\omega_R) = \frac{\lambda_R^4}{(\omega_R^2 + \lambda_R^2)^2}$$

as a weighting function, one obtains, for a Dryden spectrum, the reduced control law:

$$\tilde{K}(s) = -\left[\frac{s(s+2h')}{(s-s_0)(s-s_1)} - \frac{a_0}{\lambda_R^2}\,\frac{(s^2+2\alpha\,\nu_R s+\nu_R^2)(1+s\tau)^2(s+\lambda_R)^2}{(s-s_0)(s-s_1)(s-s_2)(1+s\tau\sqrt{3})}\right] \tag{10}$$

with: $s = i\omega_R$

and:

$$s_1 = -h' - \sqrt{h'^2 + \xi^2}$$

$$s_0 = -s_2 = -h' + \sqrt{h'^2 + \xi^2}$$

$$a_0 = \frac{2h' s_0 s_1 (1 - s_0\tau\sqrt{3})}{(s_0^2 + 2\alpha_R \nu_R s_0 + \nu_R^2)^2 (1 - s_0\tau)^2}$$

Thus the form of the control law, written in physical variables, is:

$$K(i\omega) = \frac{1}{\lambda}\left[-B_2\frac{\ell^2}{V^2}\omega^2 + i B_1\frac{\ell}{V}\omega + B_0 + \frac{C_1\frac{V}{\ell}}{i\omega - s_1\frac{V}{\ell}} + \frac{C_2\frac{V}{\ell}}{i\omega - s_2\frac{V}{\ell}} + \frac{C_3\frac{V}{\ell}}{i\omega - s_3\frac{V}{\ell}}\right] \tag{11}$$

The coefficients B_0, B_1, B_2, C_1, C_2 and C_3 do not depend on flight speed, but only on ρ, L, $C_{L,\chi}$ and $C_{m,\chi}$, and the poles are proportional to the speed. Thus the evolution of the control law with speed has been accurately determined under the adaptation characteristics of the automatic pilot.

The gain can also be expressed in explicit form, after a few mathematical manipulations. One obtains:

$$\gamma = \frac{\tilde{\sigma}_{\ddot{z}}^2}{\sigma_{\ddot{z}}^2} = \frac{\nu_R\, s_0}{2\alpha_R a_0^2}\,\psi'(\nu_R)$$

and it can be seen from this expression that the gain does not depend on flight speed.

Once the explicit form of the gain is known, it is possible to determine the effect of various parameters on a given aircraft (the Mirage III has been selected here). The effect of the scale of turbulence on gain is shown in Fig. 3: for scales above 100 m, the gain is virtually unaffected by the value of L. This is an important finding, since the scale of turbulence is a poorly understood and poorly defined parameter, and there would be problems if it had any significant effect. Fig. 4 shows the effect of altitude on gain (due to the density variation): this effect is slight. Fig. 5 shows the evolution of gain as a function of the cutoff frequency selected to compute the variance (this cutoff frequency is defined as $\lambda_R = p'\nu_R$). This evolution is very marked, and shows the importance of a physically well-founded choice of this cutoff frequency. If it is assumed that the cutoff occurs when the coherence distance is equal to the half-span b of the aircraft -- which seems to be a reasonable assumption, -- p' is found to be close to 3, which corresponds to a gain on the order of 12.

Conclusion

The above discussion has not been intended as an exhaustive investigation of the problem of active flight control in turbulence. Instead, the purpose has been, first, to show that in comparison to the closed-loop systems being developed primarily in the U.S., open-loop systems offer certain advantages in the frequency range associated with the flight mechanics of a rigid aircraft, but that they have their own limits due to the need for a representative measurement of the gust field encountered by the aircraft. Second, an attempt has been made to present the main characteristics of an open-loop system which can be adapted to current aircraft without any modifications of structure or control surfaces. Assuming the turbulence to be measured in real time, we have determined the filtering law furnishing a single control surface with commands by which it is possible to minimize the variance of aircraft response. This control law possesses poles which are proportional to the flight speed and results in a "gain" which is independent of speed and relatively insensitive to scale of turbulence or altitude.

A system of this type has just been installed on a Mirage III and is undergoing initial flight tests at this writing.

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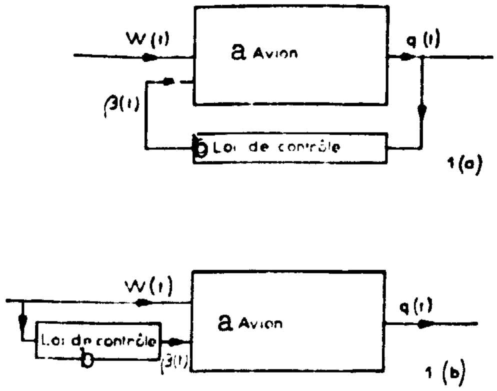

Fig. 1. Comparison of the two possible control systems.

Key: a. Airplane.   b. Control law.

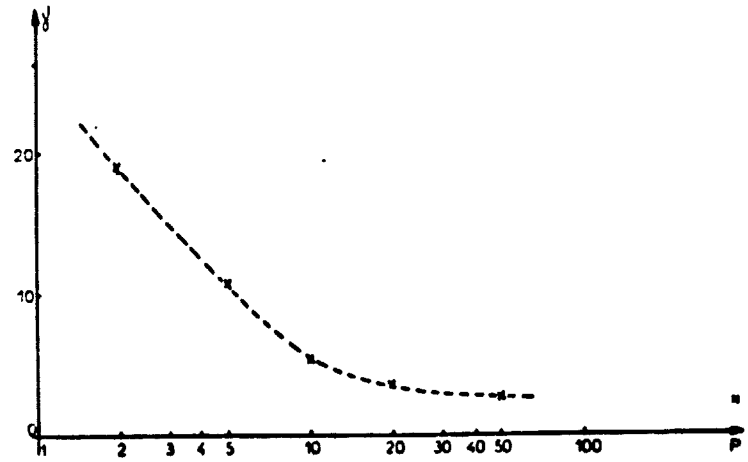

Fig. 2. Effect of filtering frequency on gain.

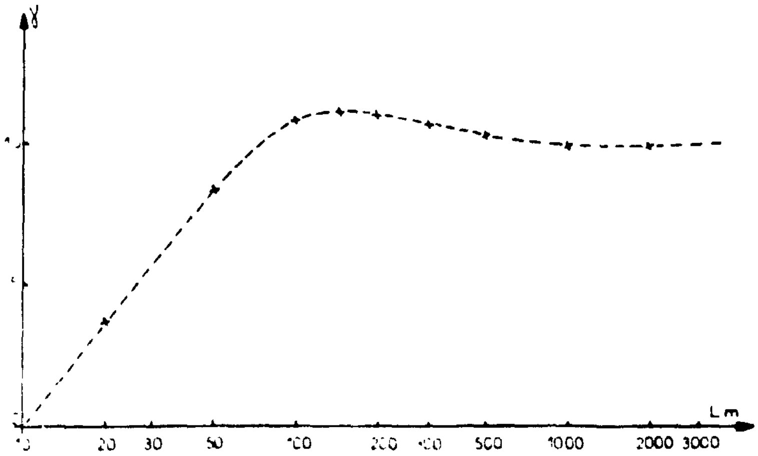

Fig. 3. Effect of scale of turbulence on gain.

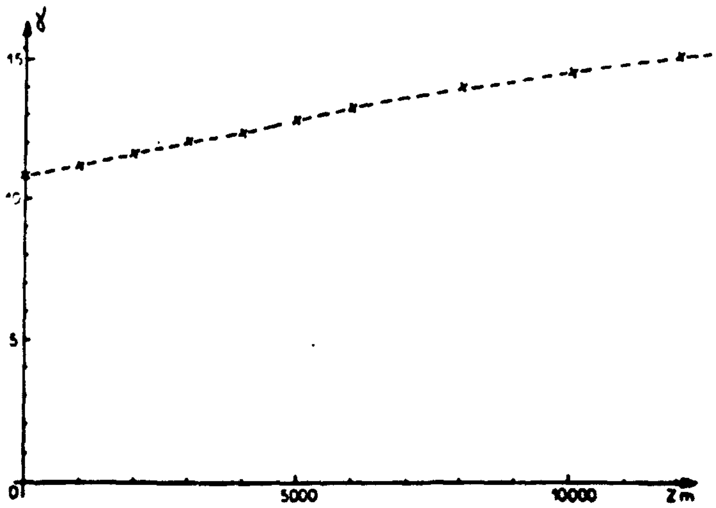

Fig. 4. Effect of altitude on gain.

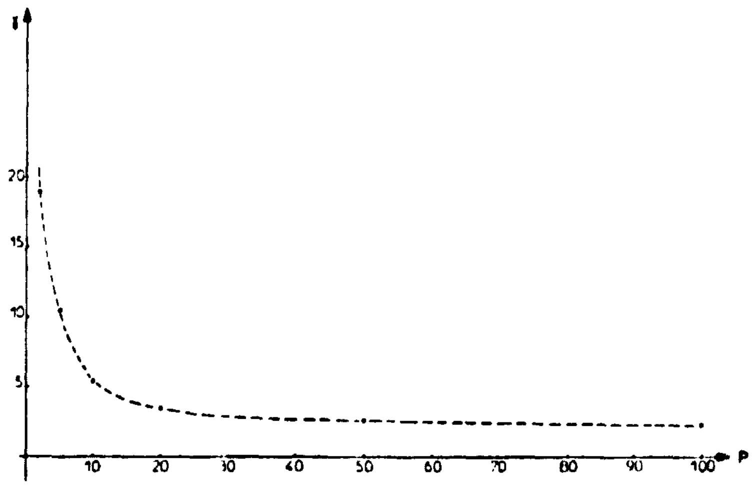

Fig. 5. Effect of cutoff frequency on gain.

TABLE 1. TYPICAL VALUES OF THE PARAMETER P

Type of Aircraft	Pitch Mode	Primary Wing Bending Mode
Caravelle	4	1
B 707	5	0.8
Concorde	10	1.3
B 747	7	1.1